
Why do we need formal (mathematical) languages?

Islam Faisal



Ambiguity in Human Languages



A mother says to her little boy.....

“Johnny, go over the road and see how old Mrs Jones is”

The boy comes back and says to his mother.....

“Mrs Jones says it is none of my business”

Ambiguity in Human Languages

“Johnny, go over the road and see **how old Mrs Jones is**”

“Johnny, go over the road and see **how old Mrs Jones is**”

Types of Ambiguity

- **Lexical ambiguity** is the presence of two or more possible meanings within a single word
 - The people are **revolting**
- **Syntactic ambiguity** is the presence of two or more possible meanings within a single sentence or sequence of words
 - Johnny's example

Why do we need propositional logical?



- Clarity and eliminate ambiguity
- Easily reason about facts
- Universal language
- Allow for automated reasoning

Aristotle's Logic

Everyone in this class will get an A.
Islam is in this class.

Islam will get an A.

Every C is A.
I is C.

I is A.

Propositional Logic

- **Predicates**: Statements that are true/false
- Example:
 - Ross loves Emily \rightarrow Love(Ross, Emily) \rightarrow L(R,E)
- Statements are made of **combinations** of predicates

What can we say?

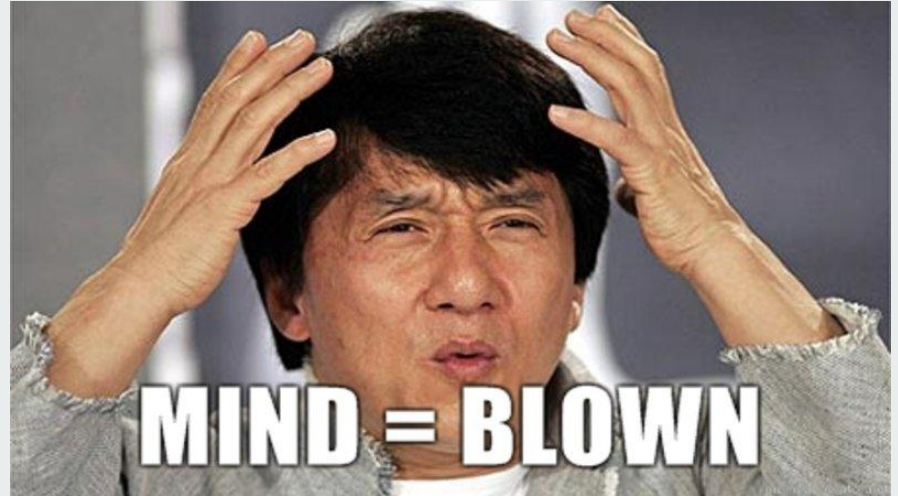
Every Student in this class will pass or fail.

$$\forall x S(x) \rightarrow P(x) \vee F(x)$$

What can **(not)** we say?

This statement is false.

- If True, it is false.
- If false, it is true.



Further Extensions

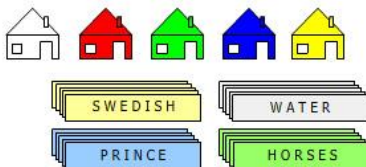


- Higher Order Logic (HOL)
- Temporal Logic
 - I ate **yesterday**
- Fuzzy Logics

Einstein Riddle

Cinematic Mode Instructions Rate: ★★★★★ Favorite Playlist

- The British man lives in a red house.
- The Swedish man keeps dogs as pets.
- The Danish man drinks tea.
- The Green house is next to, and on the left of the White house.
- The owner of the Green house drinks coffee
- The person who smokes Pall Mall rears birds.
- The owner of the Yellow house smokes Dunhill.
- The man living in the center house drinks milk
- The Norwegian lives in the first house.
- The man who smokes Blends lives next to the one who keeps cats.
- The man who keeps horses lives next to the man who smokes Dunhill.
- The man who smokes Blue Master drinks beer.
- The German smokes Prince.
- The Norwegian lives next to the blue house.
- The Blends smoker lives next to the one who drinks water.



NATION	NATION	NATION	NATION	NATION
DRINK	DRINK	DRINK	DRINK	DRINK
CIGAR	CIGAR	CIGAR	CIGAR	CIGAR
PET	PET	PET	PET	PET

```
% The Brit lives in a red house
hint1([(brit,red,_)]_).
hint1([_]T) :- hint1(T).
% The Swede keeps dogs as pets
hint2([(swede,_dog)]_).
hint2([_]T) :- hint2(T).
% The Dane drinks tea
hint3([(dane,tea,_)]_).
hint3([_]T) :- hint3(T).
% The Green house is on the left of the White house
hint4([(green,_),(white,_)]_).
hint4([_]T) :- hint4(T).
% The owner of the Green house drinks coffee.
hint5([(green,coffee,_)]_).
hint5([_]T) :- hint5(T).
% The person who smokes Pall Mall rears birds
hint6([(pallmall,bird)]_).
hint6([_]T) :- hint6(T).
% The owner of the Yellow house smokes Dunhill
hint7([(yellow,dunhill)]_).
hint7([_]T) :- hint7(T).
% The man living in the centre house drinks milk
hint8(Persons) :- person(3, Persons, (_,milk,_)).
% The Norwegian lives in the first house
hint9(Persons) :- person(1, Persons, (norwegian,_)).
% The man who smokes Blends lives next to the one who keeps cats
hint10([(blend,_),(cat)]_).
hint10([(cat),(blend)]_).
hint10([_]T) :- hint10(T).
% The man who keeps horses lives next to the man who smokes Dunhill
hint11([(dunhill,_),(horse)]_).
hint11([(horse),(dunhill)]_).
hint11([_]T) :- hint11(T).
% The man who smokes Blue Master drinks beer
hint12([(beer,bluemaster)]_).
hint12([_]T) :- hint12(T).
% The German smokes Prince
hint13([(german,prince)]_).
hint13([_]T) :- hint13(T).
% The Norwegian lives next to the blue house
hint14([(norwegian,_),(blue,_)]_).
hint14([(blue,_),(norwegian)]_).
hint14([_]T) :- hint14(T).
% The man who smokes Blends has a neighbour who drinks water
hint15([(blend,_),(water,_)]_).
hint15([(water,_),(blend)]_).
hint15([_]T) :- hint15(T).
% The question : Who owns the fish ?
question([(fish)]_).
question([_]T) :- question(T).
```

Einstein Riddle, but Formally

- (`_Man`, `_Color`, `_Drink`, `_Smoke`, `_Animal`)
- The **British** man lives in a **red** house.
→ `hint1([(brit,red,_,_,_)|_])`
- The **Swedish** Man keeps **dogs** as pets
→ `hint2([(swede,_,_,_,dog)|_])`.

Quick Recap

- Clearly defined **syntactic rules** for each language
- Not everything can be said in this language
(**semantics**)

Gödel's Incompleteness Theorem



- How good are these geeky languages?
- Good Enough, but....
- “A system cannot demonstrate its own consistency.”
~ (Gödel, 1931).



$\forall x \text{ Thank}(x)$

$\forall x \text{ Has}(x, q) \rightarrow \text{PleaseAsk}(x, q)$



References

- <https://www.youtube.com/watch?v=lw4ykgRtv3Q>
- <https://www.thoughtco.com/ambiguity-language-1692388>
- <https://baptiste-wicht.com/posts/2010/09/solve-einsteins-riddle-using-prolog.html>
- Ronnie Barker via <https://simple.wikipedia.org/wiki/Ambiguity>